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## Non Interacting Control with Internal and Input/Output Stability

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### 1. Introduction

This note deals with a number of feedback synthesis problems that appear in the context of non interacting control or (block) diagonal decoupling for finite-dimensional linear time-invariant systems. Over the past twenty-five or so years a considerable number of papers on this subject have appeared in control theory literature. For excellent overviews of the existing literature we refer to [1] or [2]. The set-up in the present paper will differ fundamentally from the one that is usually considered in the literature. We want to make clear from the outset that the purpose of this paper is *not* to present a new contribution to the "classical" problem of non interacting control as studied in the above references, but to formulate and resolve a number of *new* synthesis problems in the non interacting control context. These new synthesis problems are in principle independent of the existing problem formulations. The alternative point of view towards non interacting control as adopted in the present paper was initiated in [3], where also some preliminary results concerning the synthesis problems to be considered here can be found.

Following [3], we shall consider a plant that, apart from a control input and a measurement output (which in this paper will always be assumed to be the full state of the plant), has a given number of exogenous inputs and the same number of exogenous outputs. Basically, the problem of non interacting control that will be considered here is to design a dynamic feedback compensator from the measured plant output to the plant control input in such a way that the resulting closed loop system is block diagonal, with the sizes of the blocks compatible with the a priori given dimensions of the exogenous inputs and exogenous outputs. Stated differently: it is required to design an automatic feedback mechanism in such a way that in the closed loop system the existing interaction between the exogenous variables is eliminated and to make sure that these variables influence each other only one at a time.

The most important feature that distinguishes the above mentioned set-up from the "classical" one is that in this formulation the exogenous inputs are specified beforehand while in the classical case it is part of the problem to design these inputs. More precisely, the classical problem of non interacting control can be roughly stated as follows ([1],[2]): given a plant with a control input, a measurement output and a given number of exogenous outputs, design exogenous input variables, a precompensator having these exogenous inputs as input variables and finally a compensator from the measured output to the plant control input such that the system in closed loop is block diagonal. Additionally, in order to avoid trivialities some typical requirements on output controllability or functional reproducibility of the closed loop system are imposed. Requiring both the precompensator as well as the feedback compensator to be static then yields the so-called restricted decoupling problem, RDP [4], while allowing both compensators to be dynamic yields the extended decoupling problem, EDP [4] (as explained in [2]).

In our opinion both of the main problem formulations as stated above are useful in the context of non interacting control design. For some reason however the former one is highly neglected in control theory literature which, in our opinion, is rather surprising as its formulation appears to be a very natural one. Therefore, in [5] an extensive treatment of the former problem formulation was developed, including several stability issues. Moreover, also the natural extension of the problem to the

context of *almost* block diagonal decoupling was treated there. In the latter problem the off-diagonal blocks are not required to be exactly identically equal to zero but can be made arbitrarily small in some appropriate norm.

The purpose of this note is to summarize some of the main results from [5].

### 2. Non interacting control: problem formulation

Consider the finite-dimensional linear time-invariant system  $\Sigma$  given by

$$\dot{x}(t) = Ax(t) + Bu(t) + \sum_{i=1}^k G_i v_i(t), \quad (2.1a)$$

$$z_i(t) = D_i x(t), \quad i \in \underline{k}, \quad (2.1b)$$

with  $x(t) \in \mathbb{R}^n =: X$  the state of the system,  $u(t) \in \mathbb{R}^m =: U$  the control input,  $v_i(t) \in \mathbb{R}^{q_i} =: V_i$  the  $i$ th exogenous input and  $z_i(t) \in \mathbb{R}^{p_i} =: Z_i$  the  $i$ th exogenous output.  $k$  is assumed to be an integer larger than 1 and the symbol  $\underline{k}$  denotes the set  $\{1, 2, \dots, k\}$ . In the above  $A: X \rightarrow X$ ,  $B: U \rightarrow X$  as well as  $G_i: V_i \rightarrow X$  and  $D_i: X \rightarrow Z_i$  are linear maps. As a standing assumption  $B$  will be injective.

We shall be concerned with the design of dynamic compensators  $\Sigma_c$  described by

$$\dot{w}(t) = Kw(t) + Lx(t), \quad (2.2a)$$

$$u(t) = Mw(t) + Nx(t), \quad (2.2b)$$

with  $w(t) \in \mathbb{R}^l =: W$  the state of the compensator and  $K: W \rightarrow W$ ,  $L: X \rightarrow W$ ,  $M: W \rightarrow U$  and  $N: X \rightarrow U$  linear maps. The dimension  $l$  of the state space  $W$  will be denoted by  $\dim \Sigma_c$ . The feedback interconnection of  $\Sigma$  with  $\Sigma_c$  is a system with  $(v_1, v_2, \dots, v_k)$  as its input and  $(z_1, z_2, \dots, z_k)$  as its output and is described by the equations

$$\dot{x}_e(t) = A_e x_e(t) + \sum_{i=1}^k G_{i,e} v_i(t), \quad (2.3a)$$

$$z_i(t) = D_{i,e} x_e(t), \quad i \in \underline{k}, \quad (2.3b)$$

where we have denoted

$$x_e = \begin{bmatrix} x \\ w \end{bmatrix}, \quad A_e = \begin{bmatrix} A & BN & BM \\ L & K & N \end{bmatrix},$$

$$G_{i,e} = \begin{bmatrix} G_i \\ 0 \end{bmatrix}, \quad D_{i,e} = \begin{bmatrix} D_i & 0 \end{bmatrix}.$$

We shall denote by  $T$  the transfer matrix of the closed loop system (2.3).  $T$  is equal to the composite matrix  $(T_{ij})$  where

$$T_{ij}(s) = D_{j,e}(Is - A_e)^{-1} G_{i,e}, \quad i, j \in \underline{k}, \quad (2.4)$$

represents the transfer matrix between the  $i$ th input  $v_i$  and the  $j$ th output  $z_j$ . In [3] the following problem was introduced:

**PROBLEM 1 (non interacting control)** Problem 1 is said to be solvable if there exists a compensator  $\Sigma_c$  such that  $T_{ij} = 0$  for all  $i, j \in \underline{k}$  with  $i \neq j$ .

If a compensator  $\Sigma_c$  is such that  $T_{ij} = 0$  for all  $i \neq j$  then it will be said to *achieve non interaction*. In that case the resulting closed loop transfer matrix is block diagonal:

$$T = \text{blockdiag}(T_{11}, \dots, T_{kk}).$$

An important issue here will be stability. In the sequel a

subset  $\mathcal{C}_g$  of  $\mathcal{C}$  will be called symmetric if  $\mathcal{C}_g \cap \mathcal{R} \neq \emptyset$  and if it satisfies  $\lambda \in \mathcal{C}_g \Leftrightarrow \bar{\lambda} \in \mathcal{C}_g$ . A rational matrix will be called  $\mathcal{C}_g$ -stable (or  $g$ -stable) if its poles lie in  $\mathcal{C}_g$ . If apart from non interaction we require *input/output stability* of the closed loop transfermatrix from  $(v_1, v_2, \dots, v_k)$  to  $(z_1, z_2, \dots, z_k)$  we arrive at the following problem:

**PROBLEM 2 (non interacting control with i/o stability)** Given a symmetric subset  $\mathcal{C}_g$  of  $\mathcal{C}$ , problem 2 is said to be solvable if there exists a compensator  $\Sigma_c$  that achieves non interaction such that  $T_{ii}$  is  $g$ -stable for all  $i \in \underline{k}$ .

A different stability issue is that of *internal stability* of the closed loop system. Of course, if we succeed in finding a dynamic compensator that achieves non interaction with i/o-stability, due to the presence of uncontrollable and unobservable modes this does in general not mean that the closed loop system is internally stable (in the sense that  $\sigma(A_c) \subset \mathcal{C}_g$ ). Conversely, if the system is internally stable then it will automatically be i/o-stable with respect to the same stability set. Requiring the stronger notion of internal stability we arrive at:

**PROBLEM 3 (non interacting control with internal stability)** Given a symmetric subset  $\mathcal{C}_g$  of  $\mathcal{C}$ , problem 3 is said to be solvable if there exists a compensator  $\Sigma_c$  that achieves non interaction such that  $\sigma(A_c) \subset \mathcal{C}_g$ .

In this paper, as it should, input/output stability and internal stability will be treated as two different requirements. Correspondingly we will specify two stability sets  $\mathcal{C}_f$  and  $\mathcal{C}_s$  such that  $\mathcal{C}_f \subset \mathcal{C}_s$ . Combining the two notions we shall require that the decoupled system is internally stable with respect to the stability set  $\mathcal{C}_s$  and i/o stable with respect to the stability set  $\mathcal{C}_f$ . Typically this corresponds to requiring a fast response of the to-be-controlled output variables  $(z_1, z_2, \dots, z_k)$  and allowing a slower response of the internal part of the system (see also [6] or [7]). Formalizing this we arrive at the following version of the non interacting control problem:

**PROBLEM 4 (non interacting control with i/o and internal stability)** Given two symmetric subsets  $\mathcal{C}_f \subset \mathcal{C}_s$  of  $\mathcal{C}$ , problem 4 is said to be solvable if there exists a dynamic compensator that achieves non interaction such that  $T_{ii}$  is  $f$ -stable for all  $i \in \underline{k}$  and  $\sigma(A_c) \subset \mathcal{C}_s$ .

Clearly, problems 2 and 3 above may be obtained as special cases of problem 4 by taking  $\mathcal{C}_f = \mathcal{C}_g$ ,  $\mathcal{C}_s = \mathcal{C}$  and  $\mathcal{C}_f = \mathcal{C}_s = \mathcal{C}_g$  respectively. Problem 1 requires only  $\mathcal{C}_f = \mathcal{C}_s = \mathcal{C}$ . In [5] necessary and sufficient conditions for solvability of the above problems were obtained. In the next section we will state these results. For a more detailed treatment we refer of course to [5].

### 3. Some geometric concepts

Given a system  $(A, B)$  with state space  $X$  and a subspace  $K$  of  $X$  we shall denote by  $V^*(K)$  the supremal controlled invariant subspace in  $K$ . If  $\mathcal{C}_g$  is a symmetric subset of  $\mathcal{C}$  then  $V_g^*(K)$  will denote the supremal stabilizability subspace in  $K$ , [6]. If instead of one we specify two stability sets  $\mathcal{C}_f$  and  $\mathcal{C}_s$  then  $V_f^*(K)$  and  $V_s^*(K)$  will denote the supremal stabilizability subspaces with respect to  $\mathcal{C}_f$  and  $\mathcal{C}_s$  respectively. The system  $(A, B)$  will be called  $g$ -stabilizable ( $s$ -stabilizable) if it is stabilizable with respect to  $\mathcal{C}_g$  ( $\mathcal{C}_s$ ).

Another geometric concept that is important in the context of non interacting control is the concept of *radical* [4]. Given a finite collection  $L_i : i \in \underline{r}$  of subspaces of a linear space  $X$ , its radical is defined as the subspace

$$L_0 := \bigcap_{i=1}^r (L_i \cap \bigcap_{j=1, j \neq i}^r L_j). \quad (3.1)$$

For an extensive discussion on the various properties of the radical and its application to the "extended decoupling problem" we refer to [4].

Now consider the to-be-controlled system (2.1a-b). Denote im  $G_i$  by  $G_i$ . The radical of the collection of subspaces  $G_i : i \in \underline{k}$  will be denoted by  $G_0$ . Furthermore, define

$$K := \bigcap_{j=1}^k \ker D_j, \quad K_i := \bigcap_{j=1, j \neq i}^k \ker D_j, \quad i \in \underline{k}. \quad (3.2)$$

The following theorem is the main result of this section:

**Theorem 3.1** Problem 4 is solvable if and only if  $(A, B)$  is  $s$ -stabilizable,

$$G_i \subset V_f^*(K_i) + V_s^*(K) \quad \text{for all } i \in \underline{k} \quad (3.3)$$

and

$$G_0 \subset V_s^*(K). \quad (3.4)$$

We stress that the conditions established above can indeed be checked constructively. An actual check would involve the calculation of  $k$   $f$ -stabilizability subspaces  $V_f^*(K_i)$  and of the  $s$ -stabilizability subspace  $V_s^*(K)$ . A conceptual algorithm for this is described in [4, p.114].

As already noted in section 2, the main theorem Th. 3.1 of this section immediately provides necessary and sufficient conditions for solvability of the simpler problems 1, 2 and 3:

### Corollary 3.2

- (i) Problem 3 is solvable if and only if  $(A, B)$  is  $g$ -stabilizable,  $G_i \subset V_g^*(K_i)$  for all  $i \in \underline{k}$  and  $G_0 \subset V_g^*(K)$ .
- (ii) Problem 2 is solvable if and only if  $G_i \subset V_g^*(K_i) + V^*(K)$  for all  $i \in \underline{k}$  and  $G_0 \subset V^*(K)$ .
- (iii) [3] Problem 1 is solvable if and only if  $G_i \subset V^*(K_i)$  for all  $i \in \underline{k}$  and  $G_0 \subset V^*(K)$ .

We conclude this section by noting that in certain situations it might be desirable instead of a proper compensator to design a *strictly proper* compensator  $\Sigma_c$  that achieves non interaction. Indeed, using the methods developed in [5] it can for example be shown that there exists a compensator (2.2a-b) with  $N = 0$  such that  $T_{ij} = 0$  for all  $i \neq j$  if and only if  $G_i + AG_i \subset V^*(K_i)$  for all  $i \in \underline{k}$  and  $G_0 + AG_0 \subset V^*(K)$ .

### 4. Concluding remarks

In this note we stated some of the main results from [5] and gave solvability conditions for a rather general problem in the context of non interacting control by dynamic state feedback. This problem was a problem of *exact* block diagonal decoupling with internal stability and input/output stability. As special cases we obtained conditions for solvability of the corresponding problems where only input/output stability, only internal stability and no stability was required.

If instead of requiring the off-diagonal blocks in the closed loop transfer matrix to be exactly equal to zero we only require these blocks to be arbitrarily small in some appropriate norm, we arrive at problems in the context of approximate or *almost* non interacting control. In [5] the authors also formulated and resolved the "almost" versions of the problems treated in this note. Of course, for more details and for proofs we refer to [5].

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